

**Mathematical Economics MME2/2 – 2019/2020 (sample of the final exam)**

1. A consumer has a utility function  $u(x_1, x_2) = x_1^{\frac{1}{2}} + x_2^{\frac{1}{2}}$  and the budget constraint is  $p_1 x_1 + p_2 x_2 = I$ ,  $p_1, p_2, I > 0$ . What are the Marshallian demand functions?

(1 score)

2. Solve the expenditure minimization problem

$$\begin{aligned} \min_{x_1, x_2} \quad & p_1 x_1 + p_2 x_2 \\ & u = x_1^{\frac{1}{3}} x_2^{\frac{1}{3}} \end{aligned} \quad p_1, p_2, u > 0.$$

(1 score)

3. A monopoly has a production function  $y(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{3}}$ . Solve the profit maximization problem and cost minimization problem in long run if  $p(y) = 6y^{-\frac{1}{2}}$ ,  $v_1 = 2$  and  $v_2 = 3$ .

(1 score)

4. For the technologies  $y = \min\left\{\frac{1}{4}x_1, 2x_2\right\}$  and  $y = \frac{x_1}{2} + x_2$  compute:

- a) the total cost and the cost of production of 1 unit of output,
- b) the marginal and average cost.

Assume that prices of inputs are (4, 6).

(1 score)

5. Suppose that we have two firms that face linear demand curve  $p = 200 - \frac{1}{2}(y_1 + y_2)$  and

their cost functions are  $c_1(y_1) = \frac{1}{2}y_1^2$ ,  $c_2(y_2) = 10y_2$ , respectively.

- a) Compute the Cournot equilibrium amount of output for each firm and firms' profits.
- b) If firm 2 behaves as a follower and firm 1 behaves as a leader, compute the Stackelberg equilibrium amount of output for each firm and firms' profits.

(1 score)